On the Capacity of a Pulse Position Hopped CDMA System[1]

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ABSTRACT

Pulse Position Hopped (PPH) CDMA is a new promising multiple access technique which is very well suited for short range multipath communications, and has several benefits, such as coherent reception, low transmit power and it is near-far robust. In this paper we analyze the error-correcting capability of a system employing PPH-CDMA.

INTRODUCTION

The current emphasis on constant-envelope spread-spectrum modulations has caused engineers to ignore one design which has considerable potential, namely Pulse Position Hopping (PPH). This modulation, also known as Impulse Radio Multiple Access (IRMA) and as Ultra-Wide Bandwidth (UWB) transmission, is proposed in [2], [3], and [4]. PPH transmission has several benefits, such as coherent reception, low transmit power and it is near-far robust.
In [2] the basics of the technology for generation of the narrow pulses of duration less than 1 ns and the very low spectral density, is thoroughly described. The study of the capacity of a binary pulse position modulation (PPM) IRMA system [4] shows that, it can reach an order of several thousands of active users per cell. In [5] an experimental design is described and measurements of the multipath channel is presented. We will study a slightly different modulation method in comparison to [4], namely binary on-off modulation. In this paper, we present the results of an information-theoretical analysis of a PPH code division multiple access (CDMA) system and will present a lower bound the overall effective capacity of the system in the downlink and the uplink directions, an extended analysis can be found in [1].
The remaining part of this paper is organized as follows, in Section 2 the system model is described, in Section 3 and Section 4 the effective capacity of the uplink and downlink system is estimated and in Section 5 conclusions and future work is discussed.

SYSTEM MODEL

The PPH-CDMA system is based on on-off modulation. For the transmission of the binary sequence v_0, $v_1, \dots, v_n, \dots$; $v_n \in \{0,1\}$ this modulation uses the sequence of impulses

$$s_n(t) = v_n \cdot h(t-\tau_n), \tag{1}$$

where $\tau_0 < \tau_1 < \dots < \tau_n < \dots$ are time instances for the transmission of the nth bit. We consider signaling by rectangular pulses of duration Δ

$$h(t) = \begin{cases} A_r, & -\frac{\Delta}{2} \le t \le \frac{\Delta}{2} \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

and by Gaussian pulses of duration Δ

$$h(t) = A_g e^{-\frac{t^2}{4\gamma\Delta^2}},\ -\infty < t < \infty, \tag{3}$$

where A_r and A_g are the amplitudes of the transmitted signals and γ is a parameter. Since the pulse energy is $E = \int_{-\infty}^{\infty} h^2(t)dt$, $A_r = \sqrt{E/\Delta}$ and $A_g = \frac{\sqrt{E}}{\sqrt[4]{2\gamma\pi\Delta^2}}$. On reasons, which will be clear later, we choose $\gamma = \frac{1}{9\pi}$. We note that in the case of rectangular pulses, all of the pulse energy is concentrated in the interval $(-\Delta/2, \Delta/2)$ and in the case of Gaussian pulses this interval contains more than 99% of the energy.
We consider a PPH-CDMA system with K active users, using different scenarios in the uplink and downlink transmission, respectively. Note that the total number of users that simultaneously communicate over the channel is essentially larger than K.

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For voice communication a commonly used assumption is that the voice activity factor is 40%, i.e., only 40% of the users are active in each moment [7]. In this paper we suppose that the users uses an error correcting block code. Let $R^{(k)}$ be the transmission rate (in bits/s) of the kth user, $k = 1, 2, \ldots, K$, and let $R = \sum_{k=1}^{K} R^{(k)}$ be the overall transmission rate. We consider the case when all users transmits with the same rate, i.e., $R^{(k)} = R^{(1)}$, $k = 2, 3, \ldots, K$, and $R = KR^{(1)}$. Let $\boldsymbol{u}^{(k)} = u_0^{(k)}, u_1^{(k)}, \ldots, u_{L-1}^{(k)}$; $u_l^{(k)} \in \{0, 1\}$, be the block of information of the kth user and let $\boldsymbol{v}^{(k)} = v_0^{(k)}, v_1^{(k)}, \ldots, v_{N-1}^{(k)}$; $v_n^{(k)} \in \{0, 1\}$, be the corresponding code block, then the code rate is $r = L/N$. Assuming that each user transmits code symbols with the rate $1/T$ (symbols/s) we get $R^{(1)} = r/T$.

Uplink transmission

Consider first an uplink transmission with K active users, $K \gg 1$. Suppose that the transmission time is divided in frames of duration T, $T \gg \Delta$ and that the kth user, $k = 1, 2, \ldots, K$, transmits its nth bit $v_n^{(k)}$ of the code sequence $\boldsymbol{v}^{(k)} = v_0^{(k)}, v_1^{(k)}, \ldots, v_n^{(k)}, \ldots$; $v_n^{(k)} \in \{0, 1\}$ in the time frame $nT < t < (n+1)T$. The chips in the nth frame are transmitted with on-off modulation according to

$$s_n^{(k)}(t) = v_n^{(k)} h(t - nT - \tau_n^{(k)}). \qquad (4)$$

The periodical spreading sequence defines for each of the K users the the time instance, $\tau_n^{(k)}$, of the transmission of the nth pulse within the nth frame, $0 < \tau_n^{(k)} < T$. We will suppose that for each n, the values $\tau_n^{(k)}$, k=1, 2, $\ldots$, K, can be modeled as independent identically distributed (IID) random variables uniformly distributed on the interval $(0, T)$, i.e., the different users code symbols are not synchronized. The received signal is

$$r(t) = \sum_{n=0}^{\infty} \sum_{k=1}^{K} v_n^{(k)} h(t - nT - \tau_n^{(k)} - \delta_n^{(k)}), \qquad (5)$$

where $\delta_n^{(k)}$ is the time-offset of the signal from the kth user, $\delta_n^{(k)}$ includes propagation delay, asynchronism etc. In our model we suppose that the energy of the additive white Gaussian noise (AWGN) in the pulse interval is much less than E, such that we can neglect the presence of AWGN. We also suppose that there is a perfect power control, i.e., all pulses have the same energy, independent from the user.

In the presence of synchronization errors $\epsilon_n^{(k)}$, the output of the kth demodulator is a sequence of decision statistics

$$r_n^{(k)} = \int_{-\infty}^{\infty} r(t) h(t - nT - \tau_n^{(k)} - \delta_n^{(k)} - \epsilon_n^{(k)}) dt, \qquad (6)$$
$$n = 0, 1, \ldots,$$
$$k = 1, 2, \ldots, K.$$

We suppose that $\epsilon_n^{(k)}$ is uniformly distributed on the interval $(-\zeta/2, \zeta/2)$ if the pulses are rectangular and is zero-mean Gaussian distributed with variance ζ^2, when the pulses are Gaussian shaped. If the receiver uses hard decision decoding then it makes a decision with respect to $v_n^{(k)}$

$$\hat{v}_n^{(k)} = \begin{cases} 1, & \text{if } r_n^{(k)} \geq g, \\ 0, & \text{otherwise,} \end{cases} \qquad (7)$$

where g is a threshold. In the Section 3 we will consider the effective capacity of the uplink PPH-CDMA system using soft decision decoding.

Downlink transmission

Now consider the downlink transmission. Let $\boldsymbol{v}^{(1)}, \boldsymbol{v}^{(2)}, \ldots, \boldsymbol{v}^{(K)}$ be the code sequences from the different users on the input of the transmitter of the base station and it uses the following *majority-decision* on-off modulation

$$s_n(t) = \tilde{v}_n h(t - n\alpha) \qquad (8)$$

where

$$\tilde{v}_n = \begin{cases} 1, & \text{if } \sum_{k=1}^{K} v_n^{(k)} > K/2, \\ 0, & \text{if } \sum_{k=1}^{K} v_n^{(k)} < K/2, \\ 1, & \text{with prob. } 1/2 \text{ if } \sum_{k=1}^{K} v_n^{(k)} = K/2, \\ 0, & \text{with prob. } 1/2 \text{ if } \sum_{k=1}^{K} v_n^{(k)} = K/2, \end{cases} \qquad (9)$$

and α is the time interval between neighboring pulses, i.e., the different users code symbols are synchronized. The single user downlink transmission rate is $R^{(1)} = r/\alpha$, where r is the code rate of each user, and as for the uplink transmission the overall transmission rate is $R = KR^{(1)}$. The output of the demodulator of the kth receiver is a sequence of decision statistics

$$r_n^{(k)} = \int_{-\infty}^{\infty} r(t) h(t - n\alpha - \delta_n^{(k)} - \epsilon_n^{(k)}) dt, \qquad (10)$$
$$n = 0, 1, \ldots,$$
$$k = 1, 2, \ldots, K.$$

where $\delta_n^{(k)}$ is the propagation delay and $\epsilon_n^{(k)}$ is the synchronization error distributed analogously to the uplink case. In Section 4 we consider the effective capacity of the downlink PPH-CDMA system.

EFFECTIVE CAPACITY OF THE UPLINK PPH-CDMA SYSTEM

The most interesting performance of the CDMA system is the *effective capacity*, $\tilde{C}_{up}$, measured in bits transmitted per second. The definition of the the effective capacity almost coincide with the definition of the Shannon capacity of the channel, except that it determines a maximal achievable rate of reliable communication over the channel, *conditioned that the communication system, including the coding method, is fixed.* In our case this is the maximal achievable rate for reliable communication for the CDMA uplink communication strategy described above. In this paper we estimate the most interesting bound, the lower bound for $\tilde{C}_{up}$. (The upper bound on $\tilde{C}_{up}$ does not guarantee an nonexistence of the strategies in the uplink communication which has a larger effective capacity than this upper bound.) The lower bound depends on the *effective signal-to-noise ratio* (SNR) per time unit, η_{up}, which will be defined later. To estimate the capacity of the uplink CDMA system we first calculate the mathematical expectation of the statistic $r_n^{(k)}$ (6). Supposing that $v_n^{(k)}$ are IID equiprobable binary random variables we get for the case of perfect synchronization

$$E\left[r_n^{(k)}|v_n^{(k)}\right] = \int_{-\infty}^{\infty} v_n^{(k)} h^2(t-nT-\tau_n^{(k)}-\delta_n^{(k)})dt + \sum_{k'\neq k} E\left[r_n^{(k,k')}\right], \tag{11}$$

where $r_n^{(k,k')}$ is the contribution from the k'th, $k'\neq k$, user to the output of the kth demodulator. Here the averaging is over $v_n^{(k')}$, $\tau_n^{(k)}$, $\tau_n^{(k')}$, and $\delta_n^{(k)}$ and we neglect the interference from pulses transmitted in neighboring frames. The first term in the right hand side of (11) is equal to E when a one is transmitted and 0 when a zero is transmitted. Assuming that the propagation delay for different users is approximately the same, i.e., $\delta_n^{(k)} \approx \delta_n^{(k')}$ [1], $T \gg \Delta$, and that $\tau_n^{(k)}$ are IID random variables, uniformly distributed on $(0,T)$, we get that the mathematical expectation of $r_n^{(k,k')}$ is equal to

$$E\left[r_n^{(k,k')}\right] \approx \begin{cases} \frac{E\Delta}{2T}, \\ \frac{\sqrt{2}E\Delta}{3T}, \end{cases} \tag{12}$$

for rectangular and Gaussian pulses, respectively. From (12) follows that the mathematical expectation of the decision statistics $r_n^{(k)}$ is

$$\mu_0 \stackrel{def}{=} E\left[r_n^{(k)} \mid v_n^{(k)}=0\right] \approx \begin{cases} \frac{(K-1)E\Delta}{2T} \\ \frac{(K-1)E\Delta\sqrt{2}}{3T} \end{cases} \tag{13}$$

for rectangular and Gaussian pulses, respectively, or

$$\mu_1 \stackrel{def}{=} E\left[r_n^{(k)} \mid v_n^{(k)}=1\right] \approx \begin{cases} E+\frac{(K-1)E\Delta}{2T} \\ E+\frac{(K-1)E\Delta\sqrt{2}}{3T} \end{cases} \tag{14}$$

for rectangular and Gaussian pulses, respectively, and

$$\mu = \frac{\mu_1-\mu_0}{2} = \frac{E}{2}. \tag{15}$$

The second moment of the random variable $r_n^{(k,k')}$ is

$$E\left[\mid r_n^{(k,k')} \mid^2\right] \approx \begin{cases} \frac{E^2\Delta}{3T}, \\ \frac{E^2\Delta}{3T}, \end{cases} \tag{16}$$

for rectangular and Gaussian pulses, respectively. From (12) and (16) it follows that the variance of the statistics $r_n^{(k)}$ is

$$\sigma^2 \stackrel{def}{=} var\left[r_n^{(k)}\right] \approx \begin{cases} \frac{\Delta E^2(K-1)}{3T}, \\ \frac{\Delta E^2(K-1)}{3T}, \end{cases} \tag{17}$$

for rectangular and Gaussian pulses, respectively. Now we define the effective SNR per time unit η_{up} on the input of the decision device of the uplink receiver as

$$\eta_{up} \stackrel{def}{=} \frac{1}{T}\frac{\mu^2}{2\sigma^2} = \begin{cases} \frac{3}{8\Delta(K-1)}, \\ \frac{3}{8\Delta(K-1)}, \end{cases} \tag{18}$$

for rectangular and Gaussian pulses, respectively. Equation (18) explains our choice of the parameter γ in (3). We choose γ such that the effective SNR is the same for rectangular and Gaussian pulses with the same pulse duration Δ. We define the overall effective capacity, $\tilde{C}_{up}$ for the uplink communication as the sum of the effective capacities, $\tilde{C}_{up}^{(k)}$, k=1,2, ... ,K, of the individual users, i.e., $\tilde{C}_{up} = \sum_{k=1}^{K}\tilde{C}_{up}^{(k)}$. $\tilde{C}_{up}^{(k)}$ is defined as the maximal achievable transmission rate in the uplink direction for the kth user, measured in bits per second.

Theorem 1 *The overall effective capacity of the uplink communication system is lower-bounded by the inequality*

$$\tilde{C}_{up} > \frac{\eta_{up}K}{\ln 2} = \begin{cases} \frac{3K}{8\Delta(K-1)\ln 2} \approx \frac{3}{8\Delta \ln 2}, \\ \frac{3K}{8\Delta(K-1)\ln 2} \approx \frac{3}{8\Delta\ln 2}, \end{cases} \tag{19}$$

for rectangular and Gaussian pulses, respectively. □

[1] This assumption only simplifies the analysis. In principle, the distribution/difference of $\delta_n^{(k)}$ can be taken into account, but it will not essentially change the final estimate.

Theorem 1 is proved by computing a random coding bound on the bit error probability [1],[6].

We recall that for direct sequence CDMA the overall system capacity can be estimated as $\tilde{C} \approx \frac{W}{\ln 2}$, where W is the bandwidth. Since $\Delta \sim \frac{1}{W}$, we can see that for PPH-CDMA the effective capacity is also proportional to W.

For the case when there is a synchronization error, i.e., the statistics $r_n^{(k)}$ are defined by (6), the first and second moments of the other-user interference $r_n^{(k,k')}$ will still be defined by, (12) and (16) respectively, whereas the average attenuation, $\theta_n^{(k)}$, of the contribution to the information signal $v_n^{(k)}$, due to imperfect synchronization in the decision statistics is

$$\begin{aligned}
\theta_n^{(k)} &= \begin{cases} \int_{-\zeta/2}^{\zeta/2} \frac{1}{\zeta}\left(1 - \frac{|\epsilon_n^{(k)}|}{\Delta}\right) d\epsilon_n^{(k)} \\ \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\zeta^2}} e^{-\frac{9\pi\epsilon_n^{(k)}}{8\Delta^2}} e^{-\frac{\epsilon_n^{(k)}}{2\zeta^2}} d\epsilon_n^{(k)} \end{cases} \\
&= \begin{cases} 1 - \frac{\zeta}{4\Delta}, & \text{for rectangular pulses,} \\ \frac{1}{\sqrt{1+\frac{9\pi\zeta^2}{4\Delta^2}}}, & \text{for Gaussian pulses.} \end{cases} \qquad (20)
\end{aligned}$$

Consequently the SNR and the effective capacity decreases with a factor $\left(\theta_n^{(k)}\right)^2$.

EFFECTIVE CAPACITY OF THE DOWNLINK PPH-CDMA SYSTEM

Since the users are not synchronized, the uplink transmission in a CDMA system can be modeled as asynchronous transmission. On the other hand, the downlink transmission can be organized in a synchronous manner. The corresponding transmission strategy was described in Section 2.2. Let us estimate the statistical properties of the variables $r_n^{(k)}$ (10), on the output of the kth demodulator of the synchronous downlink transmission system. The conditional mathematical expectation of $r_n^{(k)}$ given $v_n^{(k)}$ is

$$\begin{aligned}
&E\left[r_n \mid v_n^{(k)}\right] \\
&\quad = E\left[\int_{-\infty}^{\infty} \tilde{v}_n h^2(t - n\alpha - \delta_n^{(k)} - \epsilon_n^{(k)}) dt \mid v_n^{(k)}\right] \\
&\quad\quad + \sum_{n \neq n'} E[r_{n,n'}], \qquad (21)
\end{aligned}$$

where $\delta_n^{(k)}$ is the propagation delay, $\epsilon_n^{(k)}$ is the synchronization error, $r_{n,n'}$ is the contribution from the n'th, $n' \neq n$, pulses, and the averaging is over $v_{n'}$, $\delta_n^{(k)}$ and $\delta_{n'}^{(k)}$. The first term in (21) is equal to the pulse energy, E, multiplied by

$$\begin{aligned}
&E\left[\tilde{v}_n \mid v_n^{(k)} = 1\right] \\
&\quad = \begin{cases} \frac{1}{2}\binom{K-1}{K/2}\left(\frac{1}{2}\right)^{K-1} + \sum_{k'=0}^{K/2-1} \binom{K-1}{k'}\left(\frac{1}{2}\right)^{K-1}, \\ \sum_{k'=0}^{(K-1)/2} \binom{K-1}{k'}\left(\frac{1}{2}\right)^{K-1}, \end{cases} \qquad (22)
\end{aligned}$$

when K is even or odd, respectively, or

$$\begin{aligned}
&E\left[\tilde{v}_n \mid v_n^{(k)} = 0\right] \\
&\quad = \begin{cases} \frac{1}{2}\binom{K-1}{K/2}\left(\frac{1}{2}\right)^{K-1} + \sum_{k'=0}^{K/2-2} \binom{K-1}{k'}\left(\frac{1}{2}\right)^{K-1}, \\ \sum_{k'=0}^{(K-3)/2} \binom{K-1}{k'}\left(\frac{1}{2}\right)^{K-1}, \end{cases} \qquad (23)
\end{aligned}$$

when K is even or odd, respectively. Assuming perfect synchronization and that the delay, $\delta_n^{(k)}$, is not changed during the reception, we get

$$\begin{aligned}
&E[r_{n,n'}] \\
&\quad = \frac{E}{2} \cdot \begin{cases} 1 - \frac{|n-n'|\alpha}{\Delta}, & 1 \leq |\, n - n' \,| \leq \nu, \\ 0, & \text{otherwise,} \\ e^{-\frac{9\pi\alpha^2(n-n')^2}{8\Delta^2}}, \end{cases} \qquad (24)
\end{aligned}$$

for rectangular and Gaussian pulses, respectively, and $\nu \stackrel{def}{=} \lfloor \Delta/\alpha \rfloor$, where $\lfloor\cdot\rfloor$ means integer part, and the second moment is

$$\begin{aligned}
&E\left[|\, r_{n,n'} \,|^2\right] \\
&\quad = \frac{E^2}{2} \begin{cases} \left(1 - \frac{|n-n'|\alpha}{\Delta}\right)^2, \; 1 \leq |\, n - n' \,| \leq= \nu, \\ 0, \text{ otherwise,} \\ e^{-\frac{9\pi\alpha^2(n-n')^2}{4\Delta^2}}, \end{cases} \qquad (25)
\end{aligned}$$

for rectangular and Gaussian pulses, respectively. To simplify further calculations for the rectangular pulses, we consider the case when Δ/α is an integer. Defining μ_0 and μ_1 analogously to the uplink system, we get

$$\mu_1 - \mu_0 = \begin{cases} E \cdot \binom{K-1}{K/2-1}\left(\frac{1}{2}\right)^{K-1}, & \text{K even,} \\ E \cdot \binom{K-1}{(K-1)/2}\left(\frac{1}{2}\right)^{K-1}, & \text{K odd.} \end{cases} \qquad (26)$$

If K is large, then using Stirling's formula we get $\mu_1 - \mu_0 \approx E \cdot \sqrt{\frac{2}{\pi K}}$. Using (24), and (25) the fact that $\tilde{v}_n$ are IID equiprobable binary random variables we get,

$$\begin{aligned}
\sigma^2 &= var\left[r_n^{(k)}\right] \\
&\leq \begin{cases} \frac{E^2}{4}\left(1 + \frac{2\Delta}{3\alpha}\right), \\ \frac{E^2}{4}\left(1 + \frac{4\Delta}{3\alpha} Q\left(\sqrt{\frac{9\pi\alpha^2}{2\Delta^2}}\right)\right), \end{cases} \qquad (27)
\end{aligned}$$

for rectangular and Gaussian pulses, respectively, and $Q(x)$ is defined by

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt. \qquad (28)$$

The effective SNR per time unit, c.f. (18), is

$$\begin{aligned} \eta_{down} &\stackrel{def}{=} \frac{1}{\alpha} \frac{\mu^2}{2\sigma^2} \\ &= \begin{cases} \frac{1}{\pi K\left(\alpha + \frac{2}{3}\Delta\right)}, \\ \frac{1}{\pi K\left(\alpha + \frac{4}{3}\Delta Q\left(\sqrt{\frac{9\pi\alpha^2}{2\Delta^2}}\right)\right)}, \end{cases} \qquad (29) \end{aligned}$$

for rectangular and Gaussian pulses, respectively. When α/Δ is small $Q\left(\sqrt{\frac{9\pi\alpha^2}{2\Delta^2}}\right) \approx Q(0) = 1/2$, η_{down} becomes the same for rectangular and Gaussian pulses. Analogously to the uplink, we define the effective capacity $\tilde{C}_{down}$ as the maximum achievable transmission rate in the downlink direction and we get the following theorem.

Theorem 2 *The overall effective capacity of the downlink communication system is lower bounded by the inequality*

$$\begin{aligned} \tilde{C}_{down} &> \frac{\eta_{down} K}{\ln 2} \\ &= \begin{cases} \frac{1}{\pi\left(\alpha + \frac{2}{3}\Delta\right)\ln 2}, \\ \frac{1}{\pi\left(\alpha + \frac{4}{3}\Delta Q\left(\sqrt{\frac{9\pi\alpha^2}{2\Delta^2}}\right)\right)\ln 2}, \end{cases} \qquad (30) \end{aligned}$$

for rectangular and Gaussian pulses, respectively. □

The proof of Theorem 2 is analogous to the proof of Theorem 1.
For both rectangular and Gaussian pulses the effective capacity of the downlink transmission, $\tilde{C}_{down}$, is equal to the effective capacity of the uplink, $\tilde{C}_{up}$, when $\alpha = \frac{8-2\pi}{3\pi}\Delta \approx 0.18\Delta$. Letting α go to zero the synchronous downlink system outperforms the asynchronous uplink system with up to a factor of $\frac{4}{\pi}$. The degradation of the effective capacity due to imperfect synchronization is the same as for the uplink system, i.e., the average attenuation is given by (20).

TRANSMISSION AND RECEPTION — NUMERICAL EVALUATION

We investigate the transmission in the PPH-CDMA system using, in both links, concatenated coding with an inner first order Reed-Muller code and an outer rate r_o=1/2 convolutional code. The encoder structure is described in Figure 1. In the uplink the users transmits their code symbols, $v_n^{(k)}$, according to (4)

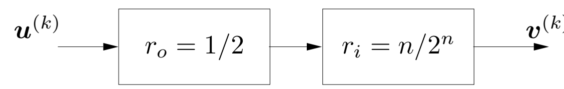

Figure 1: Encoder structure

and in the downlink the base station transmits according to the previously described majority coding rule (9). The decoding is performed in two steps. In the first step length 2^n blocks, of the received symbols are correlated with all possible inner code words, using the fast Hadamard transform (FHT) [7]. In the second step the outer convolutional decoder is fed with the soft output blocks of the FHT, of length n, and performs a Viterbi decoding producing the estimated information symbols.
Simulations indicate that using the rate $r_o = 1/2$, memory $m = 6$, (155,117) convolutional code concatenated with the rate $r_i = 8/256$ first order Reed-Muller code in the uplink, will give a bit error rate below 10^{-4}. In the downlink with $\alpha = 0.18\Delta$ we used the rate $r_o = 1/2$, memory $m = 8$, (753,561) convolutional code, as outer code and the rate $r_i = 5/32$ first order Reed-Muller code as inner code and we got also a bit error rate below 10^{-4}.

CONCLUSIONS AND FUTURE RESEARCH

The purpose of this paper was to investigate the error-correcting capability of a new CDMA scheme, PPH-CDMA. Although the technical aspects of this new method needs additional investigation, it is no doubt that PPH-CDMA is a serious candidate as a principle of multiple access communication. It seems that the downlink and the uplink in a PPH-CDMA system should be organized according to different scenarios, synchronous manner in the downlink direction and asynchronous manner in the uplink direction. The coding aspects of the PPH-CDMA communication needs additional investigation.

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