Multi-Antenna Receiver Principles for Correlated Rayleigh Channels†

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ABSTRACT

Optimal (in the sense of minimum error probability) diversity reception of single symbols for fading, noisy channels is too complex for practical implementation. In this paper, a simplified, near-optimum receiver is proposed, which is based on the statistics of the fading channel. This receiver is then analyzed by exact error probability calculations.

When there is a spread of the direction of arrival (DOA) of the incident radio waves, the proposed detector significantly gains over an adaptive antenna array, which forms a weighted sum of the received antenna signals. The whole procedure to form a weighted sum is, in fact, shown to be clearly suboptimal, no matter how the antenna weights are chosen. This is due to a loss in diversity effect for adaptive arrays.

INTRODUCTION

For simplicity, and without loss of generality, we assume that binary signaling is used under equal a priori probability, i.e., the transmitter communicates one of two different equally likely modulator waveforms \( s_i(t), i \in \{0,1\} \). One of these two waveforms is then transmitted over a channel that comprises \( D \) diversity links, i.e., there exist \( D \) ways in which the transmitted waveform can reach the receiver. In the sequel we focus on space diversity, i.e., the \( D \) diversity links are represented by means of \( D \) receiver antenna elements. In each link the transmitted waveform is perturbed by two time-varying random processes, one is the fading, which is multiplicative in nature, and the other is the noise, which is additive. Thus, the equivalent lowpass received signal becomes [1]:

\[
\mathbf{r}(t) = [r_1(t) \cdots r_D(t)]^T = s_i(t)\mathbf{f}(t) + \mathbf{n}(t), \tag{1}
\]

where superscript \( T \) denotes the transpose, \( \mathbf{f}(t) \) will be referred to as the multipath fading vector random process, or simply the fading vector, and \( \mathbf{n}(t) \) is the additive noise vector random process, or the noise vector.

The statistics of these two vector random processes will be described below. Note that both \( \mathbf{f}(t) \) and \( \mathbf{n}(t) \) are complex-valued. We shall assume that the noise vector and the modulator waveforms are statistically independent, and also that the noise and fading vectors are statistically independent. Let us further suppose for simplicity that the noise vector is composed of equally strong, independent, stationary, white Gaussian noise processes. Hence, we may write the noise covariance-function matrix as:

\[
E\{\mathbf{n}(t)\mathbf{n}^H(u)\} = N_0 \mathbf{I} \delta(t-u), \tag{2}
\]

where superscript \( H \) denotes the Hermitian (complex conjugate) transpose, \( \mathbf{I} \) the identity matrix, and \( \delta(\cdot) \) is Dirac’s delta function.

In this paper, a space-time-selective model proposed by Raleigh et. al. will be used [2]. Rayleigh-fading is assumed, i.e., the fading vector is zero-mean and Gaussian. The channel is further assumed to be frequency-nonselective (or frequency-flat).

Let \( (\rho_k, \varphi_k) \) represent the polar coordinates for receiver element number \( k \). The DOA is modeled as uniformly distributed over \( [-\Delta + \theta_0, \Delta + \theta_0] \), where \( \theta_0 \) denotes the mean DOA and \( 2\Delta \) the angle spread (AS). The DOA (as well as \( \varphi_k \)) is measured counter-clockwise from the x-axis. The fading covariance-function matrix is shown to equal:

\[
\mathbf{K}_f(t,u)^\Delta = E\{\mathbf{f}(t)\mathbf{f}^H(u)\} = J_0(\omega_m |t-u|)\mathbf{C}, \tag{3}
\]

where \( J_0(\cdot) \) is the zero-order Bessel function of the first kind, \( \omega_m \) is the maximum Doppler angular frequency shift, and \( \mathbf{C} \) is the array spatial correlation matrix. This last factor is defined as:

\[
\mathbf{C}^\Delta = \frac{1}{2\Delta} \int_{-\Delta+\theta_0}^{\Delta+\theta_0} \mathbf{a}(\theta)\mathbf{a}^H(\theta) d\theta, \tag{4}
\]

where \( \mathbf{a}(\theta) \) is the array response vector:

\[
\mathbf{a}(\theta) = [e^{j\kappa \rho_0 \cos(\theta-\varphi_1)} \cdots e^{j\kappa \rho_0 \cos(\theta-\varphi_D)}]^T. \tag{5}
\]

\( \kappa \) in (5) denotes the wave number.
OPTIMUM DIVERSITY RECEPTION

Our detection problem is a classical one, i.e. discrimination of two Gaussian signals in additive white Gaussian noise (AWGN) [3], [4]. In order to use the classical method of a likelihood ratio test (LRT) [5], the continuous-time received process must first be “represented” by a finite sequence of random variables. This transition from continuous time to discrete time will be referred to as discretization. Let us store the observables (the random numbers) in a column-vector:

\[ r = [r_1 \cdots r_N]^T. \]  

(6)

Either we could sample to obtain the representing sequence, or make an orthogonal expansion of the process and use the expansion coefficients as the observables [3], [4]. Only a brief discussion of the optimum detection rule is given in the following (cf. [6] for a more thorough treatment).

The LRT is written on the following quadratic form [5], [7]:

\[ r^H \left( K_{r,1}^{-1} - K_{r,0}^{-1} \right) r \geq \ln \left( \frac{K_{r,0}}{K_{r,1}} \right), \]

(7)

where the conditional covariance matrix is defined as follows:

\[ K_{r,i} = \mathbb{E} \left\{ r r^H | s_i(t) \right\}, \quad i \in \{0,1\}. \]

(8)

It is well known that an optimum detector structure can be found by manipulating (7) into a form so that we can let \( N \to \infty \) [5]. In the absence of additive noise (i.e. when \( n(t) \equiv 0 \), \( r \) converges to \( r(t) \) in a mean-square sense when \( N \to \infty \). We are, however, interested in the received process when it is corrupted by additive noise, and the representation is not then valid. However, in consequence of the theorem of irrelevance [8], it can be shown that \( r \) (in the limit) does preserve all of the relevant information contained in the received process. Initially there was some criticism on the lack of convergence proofs for the discrete-time representations, but [3] put the preceding work on a more rigorous basis. These calculations, then, lead to a detector structure that involves time-varying filters with impulse responses that require solutions to integral equations based on the fading channel statistics (solutions that in general cannot be found on a closed form).

In order to keep the complexity of the receiver at a more moderate level, we will not increase the number of observables beyond a finite limit (i.e. a fixed \( N \)), and under this restriction (7) still constitutes an optimum decision rule. From a purely intuitive point of view, the optimum set of \( N \) observables should be uncorrelated, and this should hold for any \( N \). Otherwise, the number of observables would be effectively less than \( N \). Karhunen-Loève expansion (KLE) is optimal in this sense, because it leads to uncorrelated observables [5].

One natural representation of the KLE is to use vector eigenfunctions and scalar eigenvalues [5], i.e., compute the observables according to:

\[ r_n = \int_0^{T_f} r^T(t) \Phi_n^*(t)dt, \quad n = 1, \ldots, N, \]

(9)

where superscript \( * \) denotes the complex conjugate, \([0,T_f]\) is the signaling interval, and the vector eigenfunctions \( \Phi_n(t) \) are chosen to satisfy a matrix-valued Fredholm equation of the second kind [5]. This discretization scheme will be referred to as VKLE, where the \( V \) is to remind us that a vector random process is expanded, and that the eigenfunctions are vector-valued.

A quandary comes up here, since the kernel of the Fredholm equation, i.e. \( s(t)s(u)G(\cos[t-u])C \), involves the transmitted waveform, which of course is unknown to the receiver. Kadota, however, has shown that a simultaneously orthogonal expansion of two signals is possible [9]. The problem is that his method seems hard to generalize; it is only exemplified under very special conditions on the fading covariance-function.

An alternative idea, which will be used in this paper, is treated in [10] and [6], where the two random processes are forced to occupy disjoint subspaces of the total signal space. The two subspaces should be orthogonal to each other, i.e., the basefunctions of one subspace should be orthogonal to all the basefunctions of the other subspace. Such a partitioning of the signal space is easily created by using time-orthogonal modulator waveforms. For simplicity, we will use two modulator waveforms \( s_i(t) \) that equal unity over the first and the second half of the signaling interval, respectively. Without loss of generality, assume that \( N \) is an even number. Compute half of the observables (i.e. \( N/2 \)) by projecting against \( N/2 \) basefunctions that occupy one of the two subspaces, and then, compute the rest of the observables by using \( N/2 \) basefunctions from the other subspace. For obvious reasons, it is important to choose the \( N/2 \) basefunctions in each subspace that correspond to most energy, i.e. the basefunctions that correspond to the largest eigenvalues.

In accordance with the above discussion we choose basefunctions that satisfy the following two criteria [5]:

1. \[ \int_{T_f}^{T_f} \Phi_k^H(t) \Phi_k(t) dt = \delta_{kk}, \]
2. \[ \lambda_k \Phi_k(t) = \int_{T_i}^{T_f} \int_0^t \Phi_k(u) \Phi_k(t) du. \]

(10)
where \( \delta_0 \) in (10) denotes the Kronecker delta, and the limits of the integrals \( \{T_f, T_r\} \) equal \( \{0, T_f / 2\} \) for the first \( N / 2 \) observables, and \( \{T_f / 2, T_f\} \) for the last \( N / 2 \) observables. Now, by applying Mercer’s theorem [5] it is straightforward to show that the conditional covariance matrices in (7) and (8) become diagonal and that the right member of (7) reduces to a zero threshold.

It is a serious fact that the basefunctions could not be analytically derived from (11). Numerical methods may be applied, but note that we have to resolve for the basefunctions whenever the kernel changes, i.e., whenever the fading parameters (the DOA, the AS, the Doppler frequency shift) change. Therefore, we would like to find some suboptimum scheme to compute the observables; a scheme that does not require the solution of (11).

**SUBOPTIMUM DIVERSITY RECEPTION**

Let us compute \( N / D \) observables in each antenna element \( k = 1, \ldots, D \):

\[
\mathbf{r}_k = \begin{bmatrix}
\mathbf{n}_{1, k} \\
\vdots \\
\mathbf{n}_{N, k/N}
\end{bmatrix} = \int_0^{T_f} \mathbf{n}_k(t) \begin{bmatrix}
\phi_{1, k}(t) \\
\vdots \\
\phi_{N, k/N}(t)
\end{bmatrix} dt = \int_0^{T_f} \mathbf{n}_k(t) \Phi_k(t) dt.
\]  

(12)

Naturally, these \( N \) random variables are stored as follows:

\[
\mathbf{r} = [\mathbf{r}_1^T \cdots \mathbf{r}_D^T]^T.
\]  

(13)

Let us use the same set of real-valued basefunctions in all antennas (i.e. we let \( \phi_{k, p}(t) = \phi_p(t) \) for all \( k \)). Now, by performing a KLE (this time the kernel is scalar-valued) and applying Mercer’s theorem [5], it can be shown that the two determinants in (7) are equal. Note, however, that the conditional covariance matrices are no longer diagonal, i.e., the observables obtained in different antenna elements are no longer independent. The described discretization will be called Sub-KLE.

For a single-antenna receiver, various simple sets of orthonormal (ON) basefunctions have been proved to give almost the same performance as basefunctions that satisfy the Fredholm equation [6]. An ON set that we will take into consideration is specified as follows: let the \( k \)th basefunction have constant amplitude \( \sqrt{N/T_r} \) over \( \{0, \ldots, k-1\}T_r/N, kT_r/N \), and let it be equal to zero elsewhere. When such a set is used, the discretization scheme will be referred to as Sub-ON.

**A WEIGHTED SUM APPROACH**

Nowadays, with the abundant progress in the signal processing area, adaptive antenna arrays are becoming increasingly popular [11], [12]. For these adaptive antennas, an ad hoc starting-point is to form a weighted sum of the antenna signals:

\[
r_{sum}(t) = \sum_{k=1}^{D} w_k r_k(t) = \sum_{k=1}^{D} w_k (s_k(t) + n_k(t)).
\]  

(14)

Derive the observables as follows:

\[
r_n = \int_0^{T_f} r_{sum}(t) \phi_n(t) dt, \quad n = 1, \ldots, N.
\]  

(15)

Suppose we receive the continuous-time sum of antenna signals in (14). Then, under this presupposition, the optimum approach is to Karhunen-Loève expand the sum (and let the number of observables go to infinity).

Much research has been pursued on deriving optimum antenna weights [11], [12]. One of the most common algorithms is the least-mean-square (LMS), which assumes that a replica of the desired signal is available as a reference signal [13]. This is of course impossible in a practical system, but will be assumed here for simplifying reasons. Simply stated, the LMS algorithm operates by aligning the phase of the signal from each antenna element with that of the reference signal, before summing the signals. Note that the array response vector in (5) contains the phase shifts for the \( D \) antenna elements, associated with a propagation path incident from an angle \( \theta \). Thus, if we suppose that there is no spread of the arrival angles of the propagation paths (i.e. \( \theta = \theta_0 \)), the LMS weights are directly found from (5):

\[
w_k = \exp[-j k \phi_k \cos(\theta_0 - \phi_k)], \quad k = 1, \ldots, D.
\]  

(16)

When the weights are chosen according to (16), the discretization is termed LMS-KLE.

The optimum combining approach, however, would be to choose weights that minimize the error probability, irrespective of the AS. This discretization technique will be referred to as ML-KLE (Maximum Likelihood KLE).

**PERFORMANCE ANALYSIS AND DISCUSSION**

An expression for the probability of error can be traced back to the fifties and the sixties, but perhaps one of the clearest derivations is due to Barrett [14]. He calculates a result that is valid for quadratic receivers in a binary hypothesis test (with zero threshold) between zero-mean complex Gaussian variables. Thus, the method directly applies to (7) (the threshold equals zero for all presented discretization schemes).

Before presenting any results, the signal-to-noise ratio (SNR) has to be defined. When the processes have equal average energies, the SNR simplifies to \( \text{SNR} = 10\log\{E_0/N_0\} \), where \( E_0 \) is the symbol energy.
Although the model does not introduce any restrictions whatsoever on the array geometry, we confine our analysis to a uniform linear array (ULA). The elements are further assumed to be located symmetric with respect to the origin, at angles $\theta = 0^\circ$ and $\theta = 180^\circ$. Moreover, the maximum Doppler shift is normalized to one tenth of the transmission rate, i.e. $\Omega_{\text{max}} = 2\pi \cdot 0.01 \cdot (1/T)$, which can be considered as fast fading (cf. [15]).

Figure 1 shows symbol error probabilities for various discretization schemes when eight observables are derived. An AS of $10^\circ$, as well as an AS of $90^\circ$, was investigated in this diagram. From figure 1, a first major observation is apparent: when there is a spread of the DOA it is clearly suboptimal to apply the LMS-KLE approach. As already pointed out, the best step we could take, under the presupposition that a weighted sum has been formed, is to Karhunen-Loève expand this sum (and let the number of observables go to infinity). However, this approach (the best possible under the given presupposition) has a poor performance compared with schemes extracting information directly from all the antenna signals (and not only from a weighted sum). The reason is that these weighted sum approaches were mainly designed for suppressing interfering signals (from interfering users or from intersymbol interference), under the assumption that no multipath fading is present.

In figure 1, it can also be seen that for a small AS, the two-element LMS-KLE array performs almost 3 dB better than a single element receiver, as expected (it performs exactly 3 dB better at zero AS). Further, it can be seen that the LMS-KLE actually performs worse at an AS of $90^\circ$, compared with an AS of $10^\circ$. This is not surprising, if we recall that the LMS algorithm was designed to align the phases of the antenna signals under the assumption of no AS.

Figure 2 shows symbol error probabilities for various discretization schemes when only four observables are derived. Just like in figure 1, the ASs $10^\circ$ and $90^\circ$ were investigated, respectively. Now, the ML-KLE scheme was also employed, i.e., the antenna weights were chosen to minimize the error probability. Since only four observables are computed in figure 2, the diversity effect is not sufficient at an AS of $10^\circ$. That is why Sub-ON and Sub-KLE do not gain at the AS $10^\circ$. One way to circumvent this problem is to increase the antenna separation, since this would lead to a decrease in correlation between the diversity links (see figure 3). The straightforward solution, however, is of course to increase the number of observables (cf. figure 1 where eight observables were used).

From figure 1 and figure 2, we conclude that diversity effect is lost when the antenna signals are summed before they are further processed.
Figure 5 shows how the error probability varies with the total number of observables. In this diagram the VKLE discretization scheme was employed, but the result is valid for the suboptimum schemes as well. $N = 10$ is, in fact, better than $N = 8$, but the characteristic bend in the curve occurs at a higher SNR (outside the range of the plot).

Moreover, the ML-KLE and the LMS-KLE were found to perform almost identically, regarding the error probability. Only at a very large AS did the ML-KLE prove to outperform the LMS-KLE, but then the gain was too small to be relevant. That these two schemes really differ is, however, most easily seen if their antenna patterns [16] are plotted (at a very large AS). In figure 6, antenna patterns are shown for a two-element array with an antenna separation of half a wavelength. The analyzed AS is 180°, and the mean DOA 60°. If unity weights are chosen (the dashed curve in figure 6), we know that this array will have maximum sensitivity for broadside reception, i.e., when the two antenna signals have the same phase. For this case twice the energy is received, i.e., the antenna pattern equals two at angles 90° and 270°. Now, consider the solid curve, which corresponds to LMS, and the stars, which correspond to ML. LMS maximizes the pattern in the DOA (as expected), whereas ML loosely speaking may be interpreted to catch as much signal energy as possible.
The proposed detector schemes are based on the covariance-function matrices. These quantities are assumed to be perfectly known to the receiver, but in a practical system they would have to be estimated, which inevitably leads to estimation errors. Thus, a highly relevant question is how sensitive these schemes are to such estimation errors. In order to address this issue, the performance of a mismatched detector was analyzed. The method for calculating the error probability referred to in the beginning of this section can be applied here as well. We found the schemes to be extremely robust to mismatches.

CONCLUSIONS

In this paper, a near-optimum simple detector was proposed for single symbol signaling over space-time-selective Rayleigh-fading channels. It could clearly be seen how crucial it is to extract information directly from all antenna signals, and not just from a weighted sum of the signals. Adaptive antenna arrays are obviously unable to fully gain the diversity effect of multipath fading channels.

Presently, the presented work has been generalized to a sophisticated modulation (minimum-shift keying) as well as to sequence detection. The goal is to make the scheme adaptively in order to exploit the inherent directivity of antenna arrays (e.g. to cancel interfering users in a multi-user environment).

REFERENCES